The Matching and Sorting of Exporting and Importing Firms: Theory and Evidence

Felipe Benguria[1]

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Abstract

This paper develops a general equilibrium model of international trade with heterogeneous exporters and heterogeneous importers. This theory is guided by new findings drawn from a matched exporter-importer dataset that characterizes the relationships between exporting and importing firms. I find that most exporters have a single importing partner, that highly productive exporters tend to trade with highly productive importers, and that the value traded is positively correlated with both exporter and importer productivities. The model analyzes the selection of exporters and importers into trading pairs and features simultaneous free entry into exporting and into importing. This theory provides a rationale for the fixed costs of entering export markets, associating them with the costs of searching for importing firms that distribute a product to final consumers abroad. The model is used to derive the implications of the matching and sorting of exporters and importers for global trade flows across sectors and destinations. I test this theory by studying the response of exporting and importing firms to the recent Colombia-U.S. free trade agreement.

[1]Assistant Professor, Department of Economics, University of Kentucky. (Email:fbe225@uky.edu Website: http://felipebenguria.weebly.com/). First version: 2013. Many thanks to James Harrigan, John McLaren, Ariell Reshef, Peter Debaere and Alan Taylor for their guidance and support, the participants of the trade research group at the University of Virginia, and seminar participants at the University of Virginia, the University of Kentucky, Duke University, the Central Bank of Chile, the Kansas Fed, the Dallas Fed, the Spring 2014 MidWest Trade Meetings, the 2014 Southern Economic Association Meetings and the 2014 Empirical Investigations in International Trade Conference at the University of Oregon.

I. Introduction

The interaction between exporting and importing firms is at the center of international markets. "International trade is between firms, not between nations," as Bertil Ohlin (1933, p.238) stated, yet canonical models of international trade based on increasing returns to scale and love of variety abstract from these interactions by assuming that exporters sell directly to final consumers in foreign markets. Also, at least since Ohlin (1933), it has been noted that only some firms export and that firm heterogeneity and market structure may be relevant to understand trade flows. Empirical patterns regarding the participation of individual firms in export markets have been documented systematically by a large literature starting with Bernard and Jensen (1995) and have motivated a new set of models (based on Melitz (2003)) that capture the implications of firm heterogeneity for aggregate trade flows and the effects of trade policy, shifting the attention of the literature to the reallocation of resources within sectors. Importing firms, on the other hand, have received less attention in this literature, but similar patterns of heterogeneity have been reported (Bernard et al (2007)). This paper studies the interaction between exporting and importing firms proposing a model to analyze the selection of exporters and importers into trading pairs. The model provides a rationale for the fixed costs of entering export markets, associating them to the costs of searching for importing firms that distribute a product to final consumers abroad. The general equilibrium model features simultaneous free entry into exporting and into importing. This theory is guided by new findings drawn from a matched exporter-importer dataset that characterizes exporting and importing firms. The model is used to derive the implications of the matching and sorting of exporters and importers for global trade flows across sectors and destinations. I test this theory by studying the response of exporting and importing firms to the recent Colombia-U.S. free trade agreement.

Section II puts forth a set of empirical observations based on a dataset that describes the transactions between exporting and importing firms. I focus on French exporters to Colombia and use records registered by Colombian Customs that report the identity of the Colombian importer and the foreign exporter in all transactions that occurred during 2010. I combine this information with balance sheet indicators on the French and Colombian firms to complete a dataset that includes a measure of the productivity of exporters and importers and the nature of the transactions between them. I find that most exporters have a single importing partner, that highly productive exporters tend to trade with highly productive importers, and that the value traded is positively correlated with both exporter and importer productivities.

Section III develops a general equilibrium model on the interaction between exporting and importing firms that is consistent with these findings and that guides the subsequent empirical analysis. In this theory, selling products to final consumers involves production and distribution. Exporting firms engaged in international trade must find distributors in foreign markets. Often, this can be a costly activity, and these search costs can be interpreted as a barrier to international trade and as a component of the fixed costs of exporting. Exporters decide how much effort to put into searching for partners in foreign markets, generating a sorting pattern between exporters and importers. The model predicts that more productive exporters search more and are matched with more productive importers and that the traded value between two firms depends positively on the productivity of both the exporter and the importer.[2] This model contributes to the literature of firm heterogeneity and trade in terms of the understanding of the fixed costs of entry into foreign markets and to the role of distribution channels in international trade. This work is closely related to a set of contemporaneous papers that study the matching between individual buyers and individual sellers across borders.[3] These subjects have been highlighted by recent surveys of this literature as areas open for further research.[4] Current understanding of fixed costs is limited: a recent survey by Bernard et al. (2012) describes them as a black box. My theory proposes that the costs of searching for partners in foreign markets are a component of these

[2]The result on the positive sorting between exporters and importers rests upon the complementarity in the production function that synthetizes the activities of exporters (manufacturers) and importers (distributors) and is consistent with the empirical findings. This assumption of the baseline model can be relaxed using a more general CES production function. This opens a more general discussion on the nature of the interaction of firms across borders. For a discussion of these topics see Milgrom and Roberts (1990), Kremer and Maskin (2006) and Grossman and Maggi (2000).

[3]Eaton et al (2012) and Monarch (2015) focus on the dynamics of the relationships between buyers and sellers. Kramarz et al (2015) study the role of customer related shocks in the volatility of french exporters. Blum et al (2012) study the role of import intermediaries (wholesalers). Kamal and Sundaram (2014) show that Bangladeshi exporters are more likely to sell to a given US importer if another firm in the same city has already done so. Sugita et al (2015) develop a Becker-type matching model of exporters and importers motivated by findings from exporter-importer data on Mexican apparel exports to the U.S. Bernard and Dhingra (2015) study the impact of trade liberalization on contract choices of exporting and importing firms, also using the Colombia U.S. FTA to test their theory. Closest to my paper, Bernard, Moxnes, and Ulltveit-Moe (2014), and Carballo, Ottavianio, and Volpe-Martincus (2013) focus on the effect of market characteristics on the number of buyers that an exporter trades with, while I focus on the sorting of exporters and importers of different types (productivities). Differently than these papers, I study the case of heterogeneous exporters and heterogeneous importers with free entry in both sides. (In these other papers, importers are heterogeneous consumers, rather than profit maximizing firms subject to free entry as in the data. A model of importing firms allows one to understand the impact of trade liberalization on these firms). Also differently from these models, my model is nested in the Melitz (2003) heterogeneous firm model, adding minimal assumptions to incorporate heterogeneous importing firms. On the empirical side, this paper is the only one that aside from observing the identity of buyers and sellers, adds additional information on their characteristics, which is essential to understand the sorting of exporters and importers.

[4]In the conclusions to his recent survey of this literature Redding (2011) suggests that an "area for further work is the microeconomic modeling of the trade costs that induce firm selection into export markets, including the role of wholesale and retail distribution networks."

fixed costs. These search costs, while still fixed with respect to production, are variable in another dimension: firms optimize their expenditure on these fixed costs. This assumption helps explain the fact that many small, less productive firms engage in international trade: they sell small amounts to small distributors.[5] The understanding of distribution networks in the context of international trade is also limited, although there is awareness that expenditure in distribution is sizable. Anderson and van Wincoop (2004) calculate that the expenditure in distribution is equivalent to a 55% ad-valorem tax. My model shows that introducing a distribution sector that stands between producers and final consumers has implications for the magnitude of trade flows and for the pattern of trade. This paper is also related to a certain extent to a recent literature on intermediaries in international trade, which is surveyed by Bernard et al (2012) and work on the role of reputation in international markets.[6]

To acknowledge search frictions when studying the matching between exporters and importers is a natural choice. In fact, Tinbergen's (1962) justification for the role of distance in the inaugural gravity equation was not only an approximation for transport costs but also a representation of information frictions.[7] Associating the search costs incurred by exporters to overcome informational barriers to the fixed cost of exporting is also an old idea that dates back until at least Glejser et al (1980). Search costs are difficult to measure, but the scarce evidence about them suggests they are relevant. Kneller and Pisu (2011) analyze the results of a survey that asks exporting firms in the United Kingdom about barriers to exporting. "Identifying who to make contact with in the first instance" and "establishing an initial dialogue with a prospective customer or business partner" are identified as trade barriers by 53.7% and 42.8% of firms in their sample of 448 firms, whereas "exchange rates and foreign currency" are mentioned as an obstacle by 41.7% of firms, and "dealing with legal, financial and tax regulations overseas", by 42.2% of these UK exporters. Empirical work regarding search frictions in international trade has been scarce. An exception is Allen (2014) who introduces information frictions in the context of the Eaton and Kortum (2002) model and, based on trade in agricultural commodities across the islands of the Philippines, concludes that the largest part of the effect of distance on trade flows is due to search costs rather than transport costs. In the appendix I extend the model to a many-country, partial equilibrium version to derive a gravity equation. This equation features both the traditional ad-valorem trade costs as well as search frictions representing the difficulties of finding partners in foreign

[5]The model of Arkolakis (2010) also features a mechanism in the same spirit, related to advertising expenditure of exporting firms.

[6]Work in the literature on intermediaries in international trade includes Antras and Costinot (2011), Blum et al (2010 and 2012), Bernard et al (2011) and Ahn et al (2011) among others. Machiavello (2010) and Machiavello and Morjaria (2015) study the importance of reputation in buyer-seller relationship in the context of wine exports from Chile and flower exports from Kenya.

[7]As Rauch (1999) argues, one reason why distance depresses trade flows is that proximity reduces search costs of learning about prices abroad, which is a necessity for differentiated products not traded in exchanges.

markets, consistent with Tinbergen's (1962) insight.

The model has implications for aggregate trade flows across destinations in a many-country interpretation of the model in partial equilibrium. The matching and sorting of exporters and importers across countries depends on the extent of search frictions. Through this channel, search frictions affect global trade flows across destinations and across sectors. As search costs rise, the search effort of exporters falls and consequently they are matched with less productive importers and sell less. This means that differently than in the Melitz (2003) model, fixed costs have an effect on aggregate trade flows through the intensive margin.[8]

To test the model, I study the response of exporting and importing firms to trade liberalization in section IV, using the recent free trade agreement between Colombia and the U.S as a natural experiment. This agreement brought large tariff cuts for U.S. exporters to Colombia. I use this episode to examine whether trade liberalization induces the reorganization of trading partnerships. According to the model, tariff concessions faced by U.S. exporters would induce them to switch to trading with more productive importers and consequently sell more (in addition to the direct effect of tariff cuts on the intensive margin). The higher profitability of the export market will lead the U.S. exporters to optimally incur in a larger search effort in order to find a more productive Colombian partner. In other words, while the marginal cost of search remains constant, its marginal benefit increases. Finding a more productive importer is more valuable in a more profitable market and trade liberalization makes paying a higher fixed cost (search cost) worth it. Comparing across industries, I find that U.S. firms in industries facing larger tariff cuts increased their exports relatively more and were more likely to switch to trading with a new partner after the agreement. Finally, I observe that U.S. exporters that switch their importing partners in response to tariff cuts switch to more productive importers, as the model suggests. Put together, this findings portrait a new mechanism at work in the response of firms to trade liberalization.

Section V concludes, proposing ideas for future research.

[8] Across sectors, the relative importance of the activity of the importers (distribution) and the exporters (production) varies. In industries where distribution is relatively unimportant compared to production, the type of importer chosen is less relevant for an exporter's sales. Consequently, the problem of searching for an importer becomes less relevant, we are in a situation closer to the Melitz (2003) model, and the effect of search costs on exporters' search intensity, revenue and profits across destinations is lower.

II. Initial Observations

In this section I put forth some simple observations drawn from a new dataset that describes the commercial relationships between exporting firms in France and importing firms in Colombia during 2010. These observations will guide the model used to study the matching and sorting of exporting and importing firms in section III.

II.A Construction of the Matched Exporter-Importer Dataset

I use a dataset that includes the detail of all export transactions and characteristics of both exporting and importing firms. The customs agency of Colombia records the universe of international transactions entering the country. The information collected includes the identity of the Colombian importer and of the foreign exporter.[9] I merge this data with balance sheet data of French and Colombian firms. I choose France as the exporting country due to its economic significance, its diversified production structure, and the fact that in France both public and private firms file financial statements.[10] A key advantage of studying the exports of France to Colombia is that French firms export a large variety of differentiated products that fit well the assumptions of a model of monopolistic competition. A disadvantage is that Colombia is not one of France's major export destinations. Similar data with the identity of exporters and importers is being used by others, but, to the best of my knowledge, this is the first paper to match these identities to additional data on these firms, including their revenue and a measure of their productivity.[11]

I define a firm in France by a "SIREN" number and a firm in Colombia by their tax ID number, "NIT". There are 958 exporting firms in France which I can identify and assign a SIREN number to and approximately 50 exporters that were not identified. Some of these were individuals rather than firms. In terms of value, the matched dataset I use represents 99.4% of the exports from France to Colombia. I keep the French exporters which report

[9] Colombian importers are identified by their name and a tax ID number. Foreign exporters into Colombia are identified by their name, city, street address, country, and telephone number (sometimes replaced by an email address). The data on foreign firms is comprehensive and well recorded. This data is available from the official statistical agency of Colombia's government (DANE).

[10] The balance sheet data of French firms includes their name and the address of every establishment, which makes the matching process with the customs data easy. It is collected by the "Greffe des Tribunaux de Commerce" (the Register of the Commerce Tribunals). It is legal information about companies collected from a government website. Other sources reporting the same data on firms' revenue do not include include the address of every establishment of each firm, which is key for matching it to the Colombian customs data.

[11] See Blum, Claro, and Horstmann (2012), Eaton et al (2012), Bernard, Moxnes, and Ulltveit-Moe (2014), and Carballo, Ottavianio, and Volpe-Martincus (2013).

Table II.1: Distribution of the Number of Importers with which an Exporter Trades, and of the Number of Exporters with which an Importer Trades.

Number of Exporters	963				
Number of Importers	950				
Importers per Exporter			Exporters per Importer		
Exporters matched to one importer	740	76.8%	Importers matched to one exporter	721	75.9%
Exporters matched to two importers	139	14.4%	Importers matched to two exporters	143	15.1%
3	41	4.3%	3	33	3.5%
4	16	1.7%	4	23	2.4%
5	10	1.0%	5	12	1.3%
6	5	0.5%	6	9	0.9%
7	3	0.2%	7	2	0.2%
8	2	0.2%	8	2	0.2%
10	1	0.1%	9	1	0.1%
11	1	0.1	12	1	0.1%
13	1	0.1%	15	1	0.1%
14	1	0.1%	21	2	0.2%
16	2	0.2%			
18	1	0.1%			

Notes: This table shows the distribution for the number of importers with which each exporter trades (left side), and the distribution for the number of exporters with which each importer trades (right side).

their revenue, which reduces the number of exporters to 858. There are 988 Colombian importers in the data initially, and 878 in the final dataset used.

II.B Most Exporters and Importers have a Single Partner

A first observation from this dataset is that the match between exporters and importers is mostly one-to-one. Table II.1 shows that out of the 963 exporters, 740 trade with only one importer, while 16 trade with 4 importers and 1 trades with 10 importers. It also shows that out of 950 importers, 721 trade with only one exporter, 23 trade with 4 exporters and 1 trades with 9 exporters.

It is also worth noting that exporters ship the large majority of the value exported to their main partners (importers). The (unweighted) average across exporters of the value shipped to their main partner is 93.3%.

II.C Who Trades with Whom

The second observation is that there is a positive correlation between the productivity of exporters and importers that trade together. I start by measuring the size of exporters and importers in terms of revenue as a proxy for their productivity. The size of French exporters is measured by their total revenue (from domestic and export sales). This information is available for 89% of the initial 963 French firms. The size of Colombian importers is measured by their revenue as well, which is available for 74% of Colombian firms since a majority but not all file financial statements[12]. Since there is a mechanical relationship between a firm's exports and another firm's imports, I also compute their revenue minus their bilateral trade. Next, I measure the productivity of exporters and importers. The model developed in the next section characterizes firms in terms of their productivities. The measurement of productivity is subject to constraints imposed by the availability of data. In the case of French exporters, I estimate a regression of revenue on a measure of cost including industry fixed effects and define productivity as the residual. The measure of cost is the expenditure on wages. In the case of the Colombian importers, I estimate the same regression and in this case the measure of cost is a balance sheet measure of labor costs. After obtaining these productivity measures, I explore who trades with whom by estimating the following equation, where each observation i corresponds to a trading pair.

$$(importer's\, productivity)_i = \beta_1 (exporter's\, productivity)_i + \phi_p + \epsilon_{ip} \tag{2.1}$$

I include two sets of industry fixed effects (ϕ_p), for the exporter's industry (based on the French industrial classification) and the importer's industry (based on the Colombian industrial classification). I consider only one relationship per exporter, the one with their most productive partner, as this will be the relevant result for the model discussed in section III. Table II.2 shows the results of the estimation of this equation. The results in the first column are obtained when using a measure of size (revenue) as a proxy for productivity. The results in the second column use their revenue minus their bilateral trade. The results in the third column correspond to using the measure of productivity described earlier.

The results indicate that there is positive assortative matching of exporters and importers. I find an economically large, positive and statistically significant coefficient with the three different measures of firm productivity. A one standard deviation increase in the exporter's productivity is associated with a .10 to .13 standard deviations increase in the productivity of the exporter's trading partner.

[12]Balance sheet data for Colombian firms is publicly available from the Colombian *Superintendencia de Sociedades*. The data includes firm's revenue, a code for its main activity, and a measure of labor costs.

Table II.2: The Productivities of Exporters and Importers in a Trading Relationship are Positively Correlated.

Dependent Variable: Importer's Productivity

	Measure of Productivity		
	Revenue	Revenue minus bilateral trade	Estimated Productivity
Exporter's Productivity	0.137***	0.125***	0.104**
	0.042	0.042	0.052
Observations	666	666	541

*Notes: The first column on the left shows the results for the estimation of equation (2.1) using firms' (log) revenue as a proxy for their productivity. The second column shows the results using their revenue minus the bilateral trade between them. The third column uses the measure of productivity described in the text. All columns include two sets of industry fixed effects, for the exporter's industry (based on the French industrial classification) and the importer's industry (based on the Colombian industrial classification) . All variables are standardized.Standard errors are reported under the estimated coefficients and are clustered by exporter and importer industry. ***. ** and * denote significance at the 1%, 5% and 10% confidence levels.*

II.D Firm-level Gravity: the Value Traded Depends on the Productivity of Both Exporters and Importers

Third, I observe that the value traded between an exporter and an importer once they have established a partnership depends on the productivity of both. I estimate the following equation.

$$log(value)_i = \beta_1(exporter's\, productivity)_i + \beta_2(importer's\, productivity)_i + \phi_p + \epsilon_{ip} \quad (2.2)$$

Each observation i corresponds to an exporter-importer pair. As before, I consider only one relationship per exporter. The independent variable is the total value traded between these firms. I include two sets of industry fixed effects (ϕ_p), for the exporter's industry (based on the French industrial classification) and the importer's industry (based on the

Colombian industrial classification) . I use the same three proxies for productivity as in the previous estimation. Table II.3 shows the results of the estimation of this equation. The estimated standardized coefficients for β_1 and β_2 are positive and statistically significant. The magnitudes of these coefficients are economically significant. A one standard deviation in the productivity of an exporter leads to a .21 to .37 standard deviations increase in exporter-importer trade. An exporter's productivity has a larger impact on trade than the importer's productivity. A one standard deviation in the productivity of the importer leads to a .13 to .16 standard deviations increase in exporter-importer trade. These results highlight the importance of considering not only exporter but also importer heterogeneity in trade models.

Table II.3: The Value Traded Depends on the Productivity of Both Exporters and Importers.

Dependent Variable: (log) Traded Value

	Measure of Productivity		
	Revenue	Revenue minus bilateral trade	Estimated Productivity
Exporter's Productivity	0.366***	0.360***	0.209***
	0.049	0.049	0.039
Importer's Productivity	0.144***	0.134***	0.161***
	0.047	0.048	0.054
Observations	666	666	541

*Notes: The first column shows the results for the estimation of equation (2.2) using firms' (log) revenue as a proxy for their productivity. The second column shows the results using revenue minus bilateral trade. The third column shows the results using the measure of productivity described in the text. All columns include two sets of industry fixed effects, for the exporter's industry (based on the French industrial classification) and the importer's industry (based on the Colombian industrial classification) . All variables are standardized.Standard errors are reported under the estimated coefficients and are clustered by exporter and importer industry. ***. ** and * denote significance at the 1%, 5% and 10% confidence levels.*

III. Theory

I consider a model with two countries, one industry, and labor as the single factor of production. Countries have identical technology and preferences, but may differ in size. Trade is a consequence of increasing returns to scale in production and love of variety preferences, as in Krugman (1979). Selling to final consumers in each country requires production and distribution. In each country, there is a set of producers who manufacture differentiated varieties. They must search for a distributor in each market they enter in order to reach the final consumers with their varieties. Both producers and distributors are heterogeneous in their productivities, as in Melitz (2003). Search is costly and is modeled as in Stigler (1961). The model features search costs as an additional barrier to trade besides the standard ad-valorem trade costs. The search costs are an interpretation of the fixed costs of production. The search process determines a probabilistic assignment of producers to distributors. The model thus generates selection of exporting and importing firms into trading pairs. I discuss the open economy case with costly trade.

III.A Consumption

Consumers in each country demand the differentiated varieties (w) produced by firms. Preferences are represented by the following utility function with constant elasticity of substitution $\epsilon = 1/(1-\alpha)$.

$$U = \left(\int_{w \epsilon \Omega} q(w)^{\alpha} dw \right)^{1/\alpha},$$

where Ω is the endogenous set of varieties consumed. The demand for a firm's variety is then the following.

$$q(w) = \frac{E}{P^{1-\epsilon}} \cdot p_f^{-\epsilon} = A \cdot p_f^{-\epsilon}, \tag{3.1}$$

where p_f is the price paid by final consumers, and the term A, a measure of the demand level determined in the equilibrium, combines the expenditure and the price index in a given market: $A = \frac{E}{P^{1-\epsilon}}$. The price index in each country is

$$P = \left(\int_{w \epsilon \Omega} p_f(w)^{1-\epsilon} dw \right)^{1/(1-\epsilon)}$$

III.B Production, Distribution and Search

Selling to final consumers requires production and distribution. Producers must search for a distributor in each market they enter. Both producers and distributors are heterogeneous in their productivities, which they draw upon entry from a probability distribution. I model the producers' search for distributors in a simple manner that resembles Stigler (1961). Searching is represented by sampling the distribution of distributors. Producers choose how much to search. A search effort of n leads to sampling the population of distributors n times (or allegorically, "meeting" n distributors). The cost of this search effort in terms of local labor is $\lambda \cdot n$, where λ is a parameter that captures search costs and varies across markets. Within this sample, producers choose the most productive distributor. Since distributors are not constrained in their capacity, a producer only needs one distributor to reach as many final consumers as he decides. By increasing the search effort, a producer meets a larger sample of distributors and, on average, ends up in a relationship with a more productive partner. Balancing this with the cost of searching is the producers' problem.[13] Distributors are passive in terms of the search process: they simply wait to be found by producers.[14]

The outcome of the search process is random. After searching n times, the type (productivity) of the distributor found and chosen is stochastic and follows a distribution which is that of the maximum within a sample of size n drawn from the population of distributors. For reasons of tractability I assume the productivities of distributors are drawn from a Fréchet distribution with shape parameter γ[15]. Mathematically, the probability of meeting and choosing a distributor of productivity θ_D is given by the following density function.[16]

$$f^n_{max}(\theta_D) = \frac{dF^{max}_n(\theta_D)}{d\theta_D} = n \cdot \gamma \cdot \theta_D^{-\gamma-1} \cdot e^{-n \cdot \theta_D^{-\gamma}}. \tag{3.2}$$

Producers manufacture differentiated varieties and operate in a context of monopolistic competition. As I have discussed, both production and distribution are necessary to sell a variety to final consumers. Producers sell the good to distributors at the wholesale price p_w which is set by the producer. Distributors take this wholesale price as part of their cost,

[13]For simplicity, the search process is modeled as simultaneous rather than sequential. Any model of search will be based on the idea that searching is costly and that a larger search effort leads to a better expected outcome (in this case, finding a more productive importer).

[14]Importers may be matched and distribute the varieties of more than one exporter. A setting with bilateral search would be an interesting extension, but should not alter the key predictions of the model.

[15]I use a Fréchet distribution for firm productivities to be able to obtain a closed form solution to the exporters' optimization problem. The model can be solved computationally with a Pareto distribution for firm productivities.

[16]The maximum out of a sample of size n drawn from a distribution $F(\theta)$ is a random variable with distribution $F^n_{max}(\theta) = \{F(\theta)\}^n$. The density function of this order statistic is calculated by taking the derivative of $\{F(\theta)\}^n$ with respect to θ. The density function of a Fréchet distribution is $f(\theta) = \gamma \cdot \theta^{-\gamma-1} \cdot e^{-\theta^{-\gamma}}$ and the distribution function is $F(\theta) = e^{-\theta^{-\gamma}}$.

which also includes the cost of the distribution activity required to reach final consumers.[17]

Distributing one unit of the good to final consumers requires $\frac{1}{\theta_D}$ units of labor for a distributor of productivity θ_D. The wage for the labor hired by producers and distributors depends on the country in which they are located.

I will use as an example in the discussion the case of a producer in Home selling to a distributor in Foreign. The unit cost function for the distributor is the following. The subscript DF describes the type of firm (a distributor) and its location (Foreign).

$$cost_{DF} = p_w^v \left(\frac{w_F}{\theta_{DF}}\right)^{1-v}. \tag{3.3}$$

This cost function corresponds to a Cobb-Douglas production function. The term v captures the relative importance of distribution.

Distributors maximize their profits by setting their final price p_f (the price charged to final consumers) equal to a constant markup over cost: $p_f = \frac{\epsilon}{\epsilon-1} \cdot cost_D$. The quantity sold is $q = A \cdot p_f^{-\epsilon}$. This is also the quantity demanded by the distributor from the producer. Given the behavior of distributors, producers face a "shifted" demand curve, which is a function of the wholesale price p_w.

I include an ad-valorem trade cost modeled in the standard "iceberg" form, such that $\tau > 1$ units of the good need to be shipped for one unit to arrive to distributors. Producers maximize their profits by choosing a wholesale price $p_w = \frac{\epsilon v}{\epsilon v - 1} \cdot \tau \cdot cost_{PH}$, where $cost_{PH} = \frac{w_H}{\theta_{PH}}$ is the unit cost for a producer of productivity θ_{PH}. This yields revenue $R_{PH} = p_w \cdot q$ and operating profits

$$\pi_{PH}^{Hf}(\theta_{PH}, \theta_{DF}) = A_F \cdot \left(\frac{\epsilon v}{\epsilon v - 1}\right)^{-\epsilon v} \cdot \left(\frac{\epsilon}{\epsilon - 1}\right)^{-\epsilon} \cdot \frac{1}{\epsilon v - 1} \cdot \tau^{1-\epsilon v} \cdot \left(\frac{w_H}{\theta_{PH}}\right)^{1-\epsilon v} \cdot \left(\frac{w_F}{\theta_{DF}}\right)^{-\epsilon(1-v)} \tag{3.4}$$

for producers.[18] The superscript Hf in this expression is used to indicate that these profits are obtained from goods produced in Home and sold in Foreign.

Knowing their operating profits $\pi_{PH}^{Hf}(\theta_{PH}, \theta_{DF})$ from a potential relationship with a distributor of productivity θ_{DF}, producers choose their optimal search effort n^*, solving:

$$max_n \int_0^\infty f_{max}^n(\theta_{DF}) \cdot \pi_{PH}^{Hf}(\theta_{PH}, \theta_{DF}) d\theta_{DF} - w_H \cdot \lambda \cdot n. \tag{3.5}$$

[17] This generates "double marginalization". See Tirole (1988, section 4.1) for a discussion of the literature on vertical relationships. Alternatives settings include bargaining between the producer and the distributor, two-part tariffs, vertical integration or vertical restraints. Ultimately, the nature of the contracts between exporters and importers or more in general between manufacturers and retailers is an empirical matter.

[18] I impose $\epsilon > 1/v$ focusing on the case in which revenue and profits are increasing in the producer's productivity, consistent with a large empirical literature.

The first term in equation 3.5 represents the expected operating profits for the producer obtained from a trading relationship with a distributor of productivity θ_{DF}, given a search effort n. The integrand is the probability of being matched with a certain distributor times the operating profits that the producer obtains from the relationship. The second term is the cost of the search effort.

The optimal search intensity of this producer in Home for a distributor in Foreign is the following[19]

$$n^*(\theta_{PH}, Z^{Hf}, \lambda \cdot w_H) = k \cdot \left(\frac{Z^{Hf} \cdot \theta_{PH}^{\epsilon \cdot v - 1}}{\lambda \cdot w_H} \right)^{\frac{\gamma}{\gamma - \epsilon \cdot (1-v)}}, \tag{3.6}$$

where $k = \left(\frac{\epsilon \cdot (1-v) \cdot \Gamma\left(\frac{\gamma - \epsilon \cdot (1-v)}{\gamma}\right)}{\gamma} \right)^{\frac{\gamma}{\gamma - \epsilon \cdot (1-v)}}$.

The term Z^{Hf} combines the wages in each country as well as the aggregate expenditure and price index in Foreign, $A_F = \frac{E_F}{P_F^{1-\epsilon}}$.

$$Z^{Hf} = A_F \cdot \left(\frac{\epsilon \cdot v}{\epsilon \cdot v - 1} \right)^{-\epsilon \cdot v} \cdot \left(\frac{\epsilon}{\epsilon - 1} \right)^{-\epsilon} \cdot \left(\frac{1}{\epsilon \cdot v - 1} \right) \cdot \tau^{1-\epsilon \cdot v} w_H^{1-\epsilon \cdot v} \cdot w_F^{-\epsilon \cdot (1-v)}$$

The operating profits of importing firms in Foreign from their relationship with a single exporter in Home are:

$$\pi_{DF}^{Hf}(\theta_{PH}, \theta_{DF}) = A_F \left(\frac{\epsilon \cdot v}{\epsilon \cdot v - 1} \right)^{1-\epsilon \cdot v} \left(\frac{\epsilon}{\epsilon - 1} \right)^{-\epsilon} \frac{1}{\epsilon - 1} \tau^{(1-\epsilon) \cdot v} \left(\frac{w_H}{\theta_{PH}} \right)^{(1-\epsilon) \cdot v} \left(\frac{w_F}{\theta_{DF}} \right)^{(1-\epsilon) \cdot (1-v)} \tag{3.7}$$

Importers may be matched and distribute the varieties of more than one exporter. The expected profits of an importer in Foreign obtained from the distribution of varieties produced in Home are:

$$\pi_{DF}^{Hf}(\theta_{DF}) = \int_0^\infty \frac{f_{max}^{n^*(\theta_{PH}, Z^{Hf}, \lambda \cdot w_H)}(\theta_{DF})}{M_{DF} \cdot f(\theta_{DF})} \cdot \pi_{DF}^{Hf}(\theta_{PH}, \theta_{DF}) \cdot M_{PH} \cdot f(\theta_{PH}) d\theta_{PH} \tag{3.8}$$

The first term in the integrand is the probability of being chosen as a trading partner by an exporter of productivity θ_{PH}. The second term stands for the profits from one such relationship. We aggregate over the distribution of exporters to obtain the total expected profits of the Foreign importer obtained from distributing home varieties.

[19] Refer to the appendix for the derivation. It is necessary to impose a large enough shape parameter $\gamma > \epsilon \cdot (1 - v)$.

III.C Equilibrium

Given wages in each country, the zero expected profit conditions obtained from the free entry assumption for producers and distributors in each country pin down the mass of firms in each of these categories. The relative wage is determined from the balanced trade condition.

The zero profit condition for Home producers states that expected profits from domestic sales and from exports are equal to the entry cost. Recall that in each term, the subscripts indicate the type of firm and its location (PH for instance stands for a producer in Home). The superscripts denote the direction of trade (Hf for instance stands for trade from Home to Foreign).

$$\bar{\pi}_{PH}^{Hh} + \bar{\pi}_{PH}^{Hf} = w_H \cdot f_{PH}^{entry} \tag{3.9}$$

The zero profit condition for Foreign producers is:

$$\bar{\pi}_{PF}^{Ff} + \bar{\pi}_{PF}^{Fh} = w_F \cdot f_{PF}^{entry} \tag{3.10}$$

To calculate the expected profits of Home producers, recall that the outcome for each producer is uncertain, depending on the success of the search process. Conditional on their productivity parameter θ_{PH}, profits from export sales $\pi_{PH}^{Hf}(\theta_{PH}, \theta_{DF})$ are distributed with density function $f_{max}^{n^*(\theta_{PH}, Z^{Hf}, \lambda \cdot w_H)}(\theta_{DF})$ and profits from domestic sales $\pi_{PH}^{Hh}(\theta_{PH}, \theta_{DH})$, with density function $f_{max}^{n^*(\theta_{PH}, Z^{Hh}, \lambda \cdot w_H)}(\theta_{DH})$,

$$\bar{\pi}_{PH}^{Hh} = \int_0^\infty \int_0^\infty \pi_{PH}^{Hh}(\theta_{PH}, \theta_{DH}) f_{max}^{n^*(\theta_{PH}, Z^{Hh}, \lambda \cdot w_H)}(\theta_{DH}) f(\theta_{PH}) d\theta_{DH} d\theta_{PH} \tag{3.11}$$

$$\bar{\pi}_{PH}^{Hf} = \int_0^\infty \int_0^\infty \pi_{PH}^{Hf}(\theta_{PH}, \theta_{DF}) f_{max}^{n^*(\theta_{PH}, Z^{Hf}, \lambda \cdot w_H)}(\theta_{DF}) f(\theta_{PH}) d\theta_{DF} d\theta_{PH} \tag{3.12}$$

Analogous expressions are obtained for the profits of Foreign producers.

Free entry into distribution leads to the zero expected profit condition that states that expected profits from marketing domestic and imported varieties are equal to the entry cost. The zero profit condition for Home and Foreign distributors are:

$$\bar{\pi}_{DH}^{Hh} + \bar{\pi}_{DH}^{Fh} = w_H f_{DH}^{entry} \tag{3.13}$$

$$\bar{\pi}_{DF}^{Ff} + \bar{\pi}_{DF}^{Hf} = w_F f_{DF}^{entry} \tag{3.14}$$

Again, an individual distributor has a stochastic outcome conditional on its productivity. We calculate an entrant's expected profits across productivities and outcomes. Entry of firms into distribution reduces the chance for each one of being found and chosen by a producer. Consider the expected profits for a Home distributor from sales of domestic varieties $\bar{\pi}_{DH}^{Hh}$ and of imported varieties $\bar{\pi}_{DH}^{Fh}$,

$$\begin{aligned}\bar{\pi}_{DH}^{Hh} &= \int_0^\infty \pi_{DH}^{Hh}(\theta_{PH}, \theta_{DH}) f(\theta_{DH}) d\theta_{DH} \\ &= \int_0^\infty \int_0^\infty \pi_{DH}^{Hh}(\theta_{PH}, \theta_{DH}) f_{max}^{n^*(\theta_{PH}, Z^{Hh}, \lambda \cdot w_H)}(\theta_{DH}) f(\theta_{PH}) d\theta_{DH} d\theta_{PH}\end{aligned} \tag{3.15}$$

$$\begin{aligned}\bar{\pi}_{DH}^{Fh} &= \int_0^\infty \pi_{DH}^{Fh}(\theta_{PF}, \theta_{DH}) f(\theta_{DH}) d\theta_{DH} \\ &= \int_0^\infty \int_0^\infty \pi_{DH}^{Fh}(\theta_{PF}, \theta_{DH}) f_{max}^{n^*(\theta_{PF}, Z^{Fh}, \lambda \cdot w_F)}(\theta_{DH}) f(\theta_{PF}) d\theta_{DH} d\theta_{PF}\end{aligned} \tag{3.16}$$

The profits of Foreign distributors are calculated under the same principles.

The price indices in each market and industry determine the level of demand. These price indices depend on the mass of producers selling in each market. The price index in the Home market is the weighted average of prices of domestic and imported varieties:

$$\begin{aligned}P_H = \Big(& M_{PH} \cdot \int_0^\infty \int_0^\infty (p_f^{Hh}(\theta_{PH}, \theta_{DH}))^{1-\epsilon} f_{max}^{n^*(\theta_{PH}, Z^{Hh}, \lambda \cdot w_H)}(\theta_{DH}) f(\theta_{PH}) d\theta_{DH} d\theta_{PH} \\ & + M_{PF} \cdot \int_0^\infty \int_0^\infty (p_f^{Fh}(\theta_{PF}, \theta_{DH}))^{1-\epsilon} f_{max}^{n^*(\theta_{PF}, Z^{Fh}, \lambda \cdot w_F)}(\theta_{DH}) f(\theta_{PF}) d\theta_{DH} d\theta_{PF} \Big)^{\frac{1}{1-\epsilon}}\end{aligned} \tag{3.17}$$

The price index in Foreign is:

$$\begin{aligned}P_F = \Big(& M_{PF} \cdot \int_0^\infty \int_0^\infty (p_f^{Ff}(\theta_{PF}, \theta_{DF}))^{1-\epsilon} f_{max}^{n^*(\theta_{PF}, Z^{Ff}, \lambda \cdot w_F)}(\theta_{DF}) f(\theta_{PF}) d\theta_{DF} d\theta_{PF} \\ & + M_{PH} \cdot \int_0^\infty \int_0^\infty (p_f^{Hf}(\theta_{PH}, \theta_{DF}))^{1-\epsilon} f_{max}^{n^*(\theta_{PH}, Z^{Hf}, \lambda \cdot w_H)}(\theta_{DF}) f(\theta_{PH}) d\theta_{DF} d\theta_{PH} \Big)^{\frac{1}{1-\epsilon}}\end{aligned} \tag{3.18}$$

Finally, the balanced trade condition determines the relative wages. The aggregate

revenue obtained by Home's exporters is equal to the aggregate revenue obtained by Foreign's exporters.

$$R_{PH}^{Hf} = R_{PF}^{Fh} \tag{3.19}$$

Again, consider that conditional on an exporter's productivity, his revenue is a stochastic outcome. The balanced trade condition is then the following:

$$\begin{aligned}\int_0^\infty \int_0^\infty & r_{PH}^{Hf}(\theta_{PH}, \theta_{DF}) \cdot f_{max}^{n^*(\theta_{PH}, Z^{Hf}, \lambda \cdot w_H)}(\theta_{DF}) \cdot f(\theta_{PH}) \cdot M_{PH} d\theta_{DF} d\theta_{PH} \\ &= \int_0^\infty \int_0^\infty r_{PF}^{Fh}(\theta_{PF}, \theta_{DH}) \cdot f_{max}^{n^*(\theta_{PF}, Z^{Fh}, \lambda \cdot w_F)}(\theta_{DH}) \cdot f(\theta_{PF}) \cdot M_{PF} d\theta_{DH} d\theta_{PF}\end{aligned} \tag{3.20}$$

III.D Sorting of Exporters and Importers

The first key result describes the sorting of producers (exporters) and distributors (importers).

Proposition I. There is positive assortative matching between exporters and importers. Higher productivity θ_P of exporters leads to

i) Higher search intensity $n^*(\theta_P)$

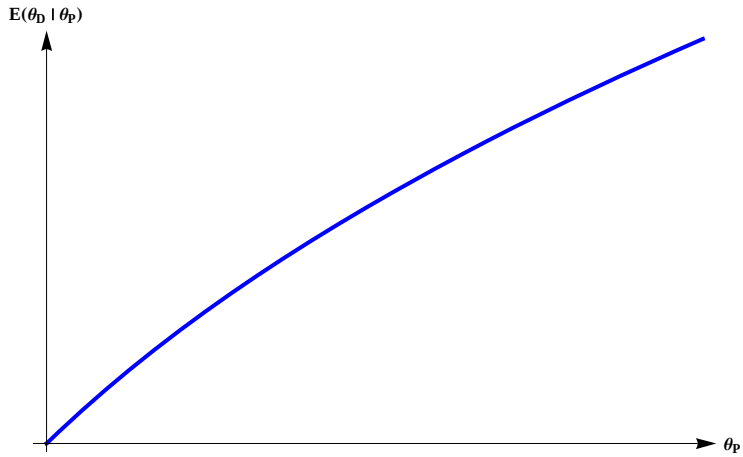
ii) Higher expected productivity $E[\theta_D|\theta_P]$ of an exporter's trading partner.

Proof: See Appendix.

More productive producers search more and thus enter partnerships with more productive distributors on average. Figure 3.1 illustrates the positive relationship between the productivity of a producer and the expected productivity of the distributor he chooses.[20] The intuition behind this result is that searching is more profitable for exporters of higher productivity, since the advantage of finding a better importer is magnified by the producer's own productivity. This is a consequence of the complementarity between exporter and importer productivities in the joint production-distribution function (equation 3.3).

[20] The outcome of the search process is random. An optimal search effort $n^*(\theta_P)$ leads to a match with an importer of expected productivity $\int_0^\infty \theta_D \cdot f_{max}^{n^*}(\theta_D) \cdot d\theta_D$. The prediction illustrated in Figure 3.1 is $\frac{d \int_0^\infty \theta_D \cdot f_{max}^{n^*}(\theta_D) \cdot d\theta_D}{d\theta_P} > 0$.

Figure III.1: The Match between Exporters and Importers



Notes: This graph represents the relationship between the productivity of an exporter of productivity θ_P (in the horizontal axis) and the expected productivity $E[\theta_D|\theta_P]$ of the importer he chooses after searching with optimal intensity $n^*(\theta_P)$.

III.E Effect of Search Costs on Sorting and Trade Flows

The second key result concerns the effect of search costs on the sorting between exporters and importers and on trade flows.

Proposition II. A decline in search costs leads exporting firms to:

i) Increase their search intensity $n^*(\theta_P)$

ii) Find and choose partners of higher expected productivity $E[\theta_D|\theta_P]$

iii) Export a larger expected value.

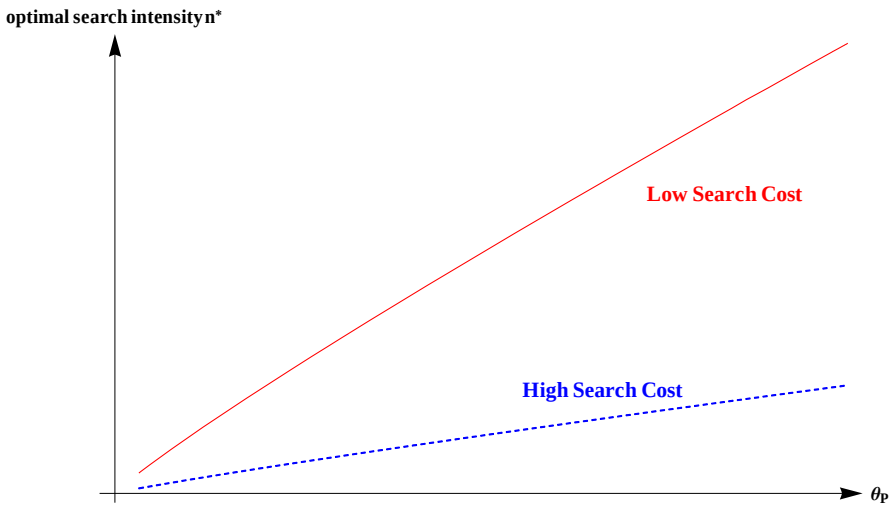
Proof: See Appendix.

As the search for importing firms becomes less expensive, exporters increase their search effort, and are matched on average to more productive importers. Figure III.2 illustrates this result. This channel through which frictions in international trade affect trade flows is additional to that of transport costs. In my model search costs are a type of fixed costs.[21] Differently than in the Melitz (2003) model, fixed costs have an effect on aggregate trade flows through the intensive margin, as shown in figure III.3.[22]

[21]Search costs are fixed costs in the sense that they are fixed with respect to the quantity produced. They are variable in a different dimension, as firms choose how much to search.

[22]The effect on the extensive margin of Melitz (2003) is generated with additional fixed cost as in section III.H.

Figure III.2: Exporters' Optimal Search Intensity



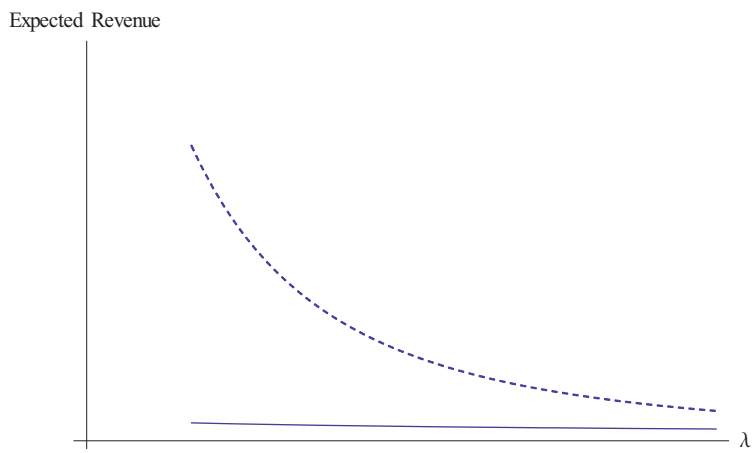
Notes: This graph represents the relationship between the productivity θ_P of exporters (in the horizontal axis) and their optimal search intensity $n^*(\theta_P)$ in the cases of high (dotted line) and low (solid line) search costs λ.

III.F Variation in Distribution Intensity

The effect of search costs on the search effort and on the revenue and profits of a producer depends on the relative importance of distribution in the combined production-distribution cost function. The intensity in distribution services is represented by v. For high values of v distribution is less relevant than production; the type of distributor chosen is less relevant for a producer's revenue. Consequently, the problem of searching for a distributor becomes less relevant, we are in a situation closer to the Melitz (2003) model, and the effect of search costs on producers' search intensity, revenue and profits is lower. This result is illustrated in Figure III.3.

Figure III.3: The Effect of Search Costs on Producers' Revenue depends on Distribution Intensity.

$v = \frac{1}{4}$ dashed line $\qquad$ $v = \frac{3}{4}$ solid line



Notes: This graph represents the relationship between search costs λ and the expected revenue of a producer of productivity θ_P (held constant in this graph). The dashed line shows the relationship for a product for which distribution is relatively important compared to production. In the solid line, distribution is unimportant, and the negative effect of search costs on the producer's revenue is lower. In other words, the dashed line has a steeper slope.

III.G "Firm-Level Gravity"

Trade flows between firms depend on the productivity of exporters and importers, generating a relationship that resembles the gravity equation for trade between countries. I term this relationship "firm-level gravity".

Proposition III. The volume of trade between two firms depends:

i) Positively on the productivity of the exporter and the importer.
ii) Negatively on ad-valorem trade costs.

Proof: By inspection of the revenue function describing the traded value between an exporting and an importing firm.[23]

[23]The revenue of a producer in Home trading with a distributor in Foreign is the following

$$R_{PH}^{Hf}(\theta_{PH}, \theta_{DF}) = A_F \left(\frac{\epsilon \cdot v}{\epsilon \cdot v - 1}\right)^{1-\epsilon \cdot v} \left(\frac{\epsilon}{\epsilon - 1}\right)^{-\epsilon} \tau^{(1-\epsilon)\cdot v} \left(\frac{w_H}{\theta_{PH}}\right)^{(1-\epsilon)\cdot v} \left(\frac{w_F}{\theta_{DF}}\right)^{(1-\epsilon)\cdot(1-v)} = Z_d \cdot \theta_{PH}^{\epsilon \cdot v - 1} \cdot \theta_{DF}^{\epsilon \cdot (1-v)}$$

In contrast, in models based on Melitz (2003) trade flows depend on the characteristics (typically productivity) of exporting firms only.

III.H Selection into export markets

The model described earlier in this section does not generate an export productivity cutoff that explains the selection of highly productive firms into export markets as in Melitz (2003). The model can be extended to include an additional fixed cost of entry into export markets (F) to generate this result. The problem for a producer in Home searching for an importer to reach final consumers in Foreign becomes:

$$max_n \int_0^\infty f^n_{max}(\theta_{DF}) \cdot \pi^{Hf}_{PH}(\theta_{PH}, \theta_{DF}) d\theta_{DF} - w_H \cdot \lambda \cdot n - w_H \cdot F \tag{3.21}$$

Firms with productivity below $\underline{\theta_{PH}} = k \cdot (\frac{F \cdot \lambda^{\delta-1}}{(Z^{Hf})^\delta})^{\frac{1}{\delta \cdot (\epsilon \cdot v - 1)}}$ do not export, where $\delta = \frac{\gamma}{\gamma - \epsilon \cdot (1-v)}$.

IV. Exporter-Importer Matches and Trade Liberalization: the U.S. - Colombia Free Trade Agreement

The U.S.-Colombia FTA provides an ideal quasinatural experiment to test the theory of exporter-importer sorting described by the model. The model predicts that by making the Colombian market more profitable. U.S. exporters find it optimal to incur in larger search costs in order to find more productive importing partners in Colombia. This reorganization of exporter-importer matches is a novel dimension on the response of firms to trade liberalization.

The tariff cuts favoring U.S. exporters to Colombia were large and there was substantial variation across industries. I design an empirical strategy taking advantage of this cross-industry variation.

The first subsection discusses the theory. Subsection IV.B provides background on the FTA, and shows that the liberalization led to an increase in U.S. exports to Colombia. Subsection IV.C discusses the matched exporter-importer data on U.S exporting firms and Colombian importing firms. The core of the analysis in subsection IV.D links the liberalization to the reorganization of exporter-importer matches, in consonance with the mechanisms in the model.

IV.A The Effect of Trade Liberalization in the Theory

The decline in trade costs (tariffs) faced by U.S. exporters is akin to an exogenous increase in demand and profitability of the Colombian market in the liberalized industries. The higher profitability of the export market will lead the U.S. exporters to find it optimally to incur a larger search effort in order to find a more productive Colombian partner. In other words, while the marginal cost of search remains constant, its marginal benefit increases. Finding a more productive importer is more valuable in a more profitable market. The liberalization makes paying a higher fixed cost (search cost) worth it. Essentially, recall that the fixed cost of exporting (search cost) is endogenous and optimized by each firm, differently than in standard heterogeneous-firms trade models.

This partial equilibrium argument is accompanied by general equilibrium results. The change in the exogenous trade cost parameter leads to an increase in the mass of exporters

in the U.S. and the mass of importers in Colombia. The test described below focuses only on the reorganization of exporter-importer matches.

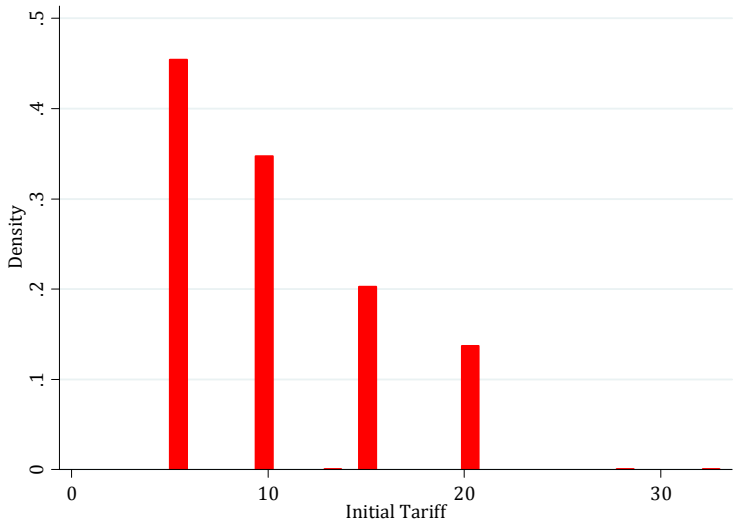
IV.B The U.S. - Colombia Free Trade Agreement

This section briefly describes the trade agreement and shows it led to an increment in U.S. exports to Colombia. The free trade agreement signed by the U.S. and Colombia reduced tariffs starting on May 15th 2012. U.S. exporters to Colombia faced large tariff cuts in many industries. Most of these were effective immediately and reduced tariffs to zero. Due to the short span of the post-liberalization period, I focus on these industries in the empirical design described in section IV.D. These tariff cuts concern a large number of U.S. firms that export to Colombia, as well as a large number of Colombian importers from the U.S.

The following figure (IV.1) illustrates the density of the tariff cuts faced by U.S. exporters. Descriptive statistics on these tariffs faced by U.S. exporters in Colombia before and after the agreement are reported in table IV.1. A first essential observation is that initial pre-FTA tariffs were large in many industries, and the reduction was also large as many of these were fully eliminated. A second important point is that there was substantial variation across industries, as shown in the figure.

The trade data show an increase in U.S. exports to Colombia immediately after the agreement. A first observation, shown in figure IV.2, is that the ratio of U.S. exports to Colombia over U.S. exports to other South American markets that did not implement this tariff cuts rose substantially. While, this pattern is evident at first sight, the figure illustrates a change in the slope of a line fitted to the trade data before and after the agreement.

FIGURE IV.1: Tariff Cuts for U.S. Exports to Colombia.



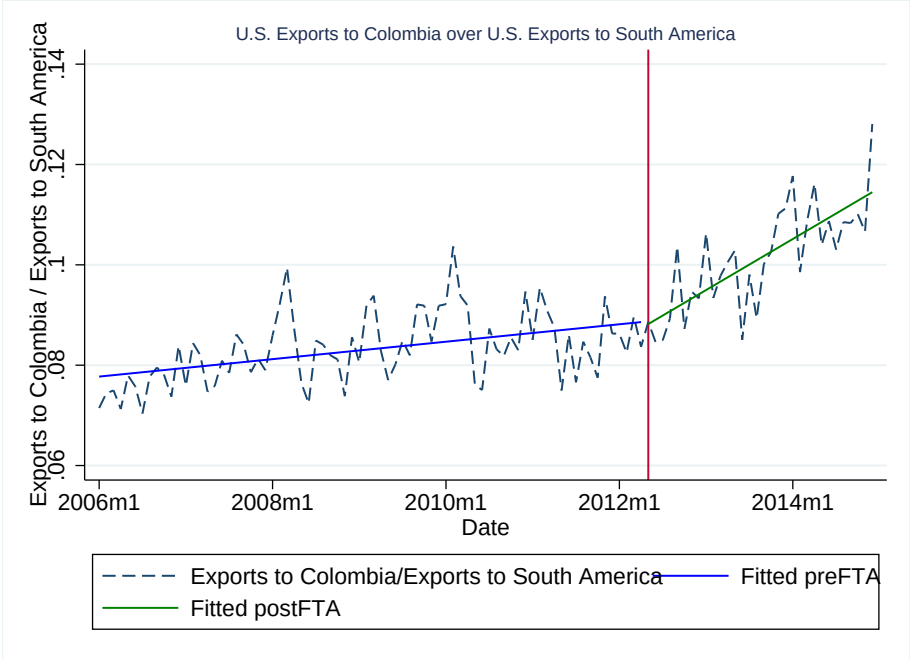
Notes: This figure shows the density of tariffs faced by U.S. exporters to Colombia according to their pre-FTA value at the HS-10 digit level. This figure is restricted to product categories that were liberalized immediately and completely.

TABLE IV.1: Descriptive Statistics for Colombian Tariffs on U.S. Exports

	PRE	POST
Mean Tariff across HS10 products	12.39588	3.45525
Median Tariff across HS10 products	15	0
Maximum Tariff across HS10 products	80	50
Standard Deviation of Tariff across HS10 products	6.664	6.596
Number of HS10 products with zero tariff	95	3619

Notes: This table reports descriptive statistics for Colombian tariffs on U.S. exported before and after the Free Trade Agreement starts in May 2012 (the POST data corresponds to one year after the start of the FTA). The data is obtained from the Free Trade Agreement's website: http://trade.gov/fta/colombia/ and http://www.tlc.gov.co/publicaciones.php?id=727.

FIGURE IV.2: The Effect of the FTA on U.S. Exports to Colombia..



Notes: This figure shows the evolution of U.S. exports to Colombia as a fraction of U.S. exports to South America at a monthly frequency during January 2006- May 2014 (blue dashed line). The green sold lines correspond to the fitted linear prediction estimated separately before May 2012 and after May 2012. Data is obtained from the U.S. Census Bureau.

To delve deeper into this relationship and establish the causality of the impact of the FTA on U.S. exports, I use product-level trade data to compare the evolution of U.S. exports in liberalized and non-liberalized sectors. I show that the Free Trade Agreement increased exports from the U.S. to Colombia following a triple difference in differences empirical design. I compare U.S. exports in liberalized industries to non-liberalized industries (first difference) before and after the agreement (second difference) to Colombia and to other South American markets (third difference). Specifically, I estimate the following gravity equation for U.S. exports to Colombia and every other South American country. I construct a categorical indicator for liberalized industries which I compare to non-liberalized industries. I define two time periods: pre-agreement (2011) and post-agreement (2013).

$$\log(\text{Exports})_{cit} = \beta_1 \cdot \text{POST}_t \cdot \text{lib}_i \cdot \text{Colombia}_c + \beta_2 \cdot \text{POST}_t \cdot \text{lib}_i + \beta_3 \cdot \text{POST}_t \cdot \text{Colombia}_c + \beta_4 \cdot \text{lib}_i \cdot \text{Colombia}_c + \epsilon_{cit} \quad (4.1)$$

In this equation, each observation is a product (i) - destination (c) - year (t) com-

bination. Products are defined at the HS-6 digit level. While the U.S. export data is disaggregated upto the HS-10 level, it can only be matched to the Colombian tariffs at the internationally-standarized HS-6 level. Robust standard errors are clustered by industry, destination and period using multiway clustering. The data on U.S. exports is obtained from the U.S. Census Bureau's "U.S. Exports of Merchandise".

Table IV.2 reports the results for equation 1. The coefficient of interest on the triple interaction is positive and economically and statistically significant. It can be interpreted as an increase in U.S. exports in liberalized industries compared to non-liberalized industries in Colombia compared to similar South American markets.

TABLE IV.2: The Effect of the FTA on U.S. Exports to Colombia.

Dependent Variable: (log) Value of U.S. Exports by Industry and Country	
$Post_t \cdot lib_i \cdot Colombia_c$	0.1332***
	0.0175
$Post_t \cdot Colombia_c$	0.0174
	0.0334
$lib_i \cdot Colombia_c$	0.0089
	0.1020
$Post_t \cdot lib_i$	-0.0115
	0.0181
Country F.E.	Yes
HS-6 Product F.E.	Yes
Time period F.E.	Yes
Observations	48039

*Notes: This table reports the results of the estimation of equation 4.1. Destination countries are Colombia, Argentina, Bolivia, Brazil, Chile, Ecuador, Paraguay, Peru, Uruguay and Venezuela. Data is obtained from the U.S. Census Bureau's "U.S. Exports of Merchandise" which reports U.S. exports disaggregated at the country - HS10 product level. Standard errors are reported under the estimated coefficients. ***. ** and * denote significance at the 1%, 5% and 10% confidence levels.*

TABLE IV.3: Descriptive Statistics for U.S. Exporters to Colombia

Exporters trading with single partner in PRE and POST	5070
In immediately liberalized industries	3565
Share switching trading partner	25%
Mean Value Exported - PRE (USD)	780 000
Mean Value Exported - POST (USD)	948 000

Notes: This table reports descriptive statistics for U.S. exporters to Colombia before and after the Free Trade Agreement starts in May 2012 (PRE corresponds to 2011/5 to 2012/6 and POST to 2013/5 to 2014/6). The first row reports the number of U.S. firms exporting to Colombia in both periods, trading with a single partner in both. The second row reports the share of those firms in industries liberalized immediately and completely, which are the ones I consider in the analysis. The third column shows the share of those firms switching partners between both periods. The last two rows show the mean value exported by these U.S. exporters in each period.

IV.C Data on U.S. Exporters and Colombian Importers

I use the information on import transactions collected by Colombian customs described in section II.A. These records contain the name of the Colombian importer and the US exporter in each international transaction. The data includes the name, address, city and telephone number or email address of the US exporters. Since I will be interested in identifying firms rather than plants, I use the names of these U.S. firms but not their address to define a firm. On the importing side, I observe the tax ID number of the Colombian importers, which define a Colombian firm. Table IV.3 presents descriptive statistics on U.S. exporters to Colombia. Additionally I use balance sheet data on these Colombian importers to observe their revenue (size) and a measure of their productivity. This data was introduced in section II.

IV.D The Free Trade Agreement and Exporter-Importer Matches.

I investigate the effect of the trade liberalization on the reorganization of U.S. exporter - Colombian importer relationships in several steps. First, I show that tariff concessions made by Colombia to U.S. exporters raised the sales of these exporters in this market. Second, I find that U.S. exporters respond to tariff cuts by switching their importing partners. Finally, I show that I show that U.S. exporters that switched importing partners in response to the tariff cuts chose larger and more productive importing partners.

IV.D.1 Tariff Cuts lead to an Increase in the Intensive Margin of Exports.

First I show that tariff cuts in favor of U.S. exporters to Colombia led to an increase in the value exported by these U.S. firms (i.e. an increase in the intensive margin of exports).

I follow a difference in differences approach, comparing the exports of U.S. firms to Colombia before and after the agreement and across industries facing different degrees of liberalization.

I measure the exposure of U.S. exporters to trade liberalization computing the tariff cut relevant to each exporting firm. I restrict the analysis to firms in sectors liberalized immediately and completely, which represent the large majority of U.S. exports to Colombia. The tariff associated to each firm is computed as a weighted average of the tariff of each of the firm's products, using product shares in the pre-agreement period as weights. In the expression below, e indexes U.S. exporters, p indexes HS-6 products. $\Delta Tariff_p$ stands for the change in ad-valorem tariffs for product p exported from the U.S. to Colombia and v_{ep}^{PRE} is the value exported by exporter e of product p during June 2011 - May 2012 (before the FTA).[24]

$$\Delta\text{Tariff}_e = \frac{\sum_p v_{ep}^{PRE} \cdot \Delta\text{Tariff}_p}{\sum_p v_{ep}^{PRE}} \tag{4.2}$$

I estimate the following equation, with the change in (log) exports of exporting firm e as the dependent variable. The independent variable of interest is the change in tariffs associated to that firm. I define the pre-liberalization period as June 2011-May 2012 and the post-liberalization period as June 2013 - May 2014.

$$\Delta\text{Exports}_e = \beta \cdot \Delta\text{Tariff}_e + \epsilon_e \tag{4.3}$$

I include industry-level fixed effects at the HS-2 digit level. This means I am comparing different industries within sectors; for instance, manufacturers of photographic cameras to manufacturers of video recorders, rather than to producers of pharmaceuticals. I cluster standard errors according to the tariff category associated to each firm.

I find that U.S. exporters facing larger tariff cuts did increase their exports to Colombia by more. A one standard deviation larger tariff reduction is associated with a 0.14 standard deviations larger revenue. The results are reported in table IV.4. Descriptive statistics are found in the appendix.

[24] I obtain the tariff data directly from the customs records of Colombias imports. These tariffs are not yet available in the World Banks TRAINS dataset. They are available in the FTA website of the Office of the U.S. Trade Representative but are reported in the Colombian classification of 2004, which is difficult to map into more recent HS codes. In the customs data, transactions report the ad-valorem tariff paid. I average these by six digit H.S. code for U.S. exports.

Table IV.4: Tariff Cuts lead to an Increase in the Intensive Margin of Exports.

Dependent Variable: $\Delta(log)\text{Exports}_e$		
	(1)	(2)
ΔTariff_e	-0.192***	-0.140***
	0.062	0.048
$(log)\text{Exports}_{e,t-1}$		-0.435***
		0.024
Industry Fixed Effects	Yes	Yes
Observations	3565	3565

*Notes: This table shows the results of the estimation of equation 4.3. All columns include industry fixed effects at the HS-2 digit level. All variables are standardized. Standard errors are reported under the estimated coefficients. Errors are clustered by sector at the HS-2 digit level. ***. ** and * denote significance at the 1%, 5% and 10% confidence levels.*

IV.D.2 Tariff Cuts lead to Switching Importing Partners.

Next, I ask whether firms facing larger tariff concessions where more likely to switch their import partners. I focus on firms which exported to Colombia both before and after the agreement. I restrict the sample to firms with a single partner before and after the liberalization. I estimate the following equation. The dependent variable $Switch_e$ takes a value of one for exporters that switch importers between June 2011 - May 2012 (the initial interval) and June 2013 - May 2014 (the final interval). The independent variables are again the change in tariffs associated to each firm, computed as indicated in equation 4.2. As before, I include HS 2-digit level industry fixed effects to compare once more across similar industries. Since I include these fixed effects I estimate a linear regression model rather than of a probit model, to avoid the incidental parameters problem.

$$\text{Switch}_e = \beta \cdot \Delta\text{Tariff}_e + \epsilon_e \tag{4.4}$$

I find that firms in industries facing larger tariff cuts were more likely to switch to a new trading partner, as shown in table IV.5. A one standard deviation larger tariff reduction is associated with a 0.072 standard deviations higher probability of switching importing partners. The results support the idea that firms have incentives to switch to trading with more productive importing partners due to the increased profitability in an export market that results from a liberalization. This idea is explored further in the next subsection.

Table IV.5: Tariff Cuts lead to Switching Importing Partners.

Dependent Variable: Switch_e		
	(1)	(2)
ΔTariff_e	-0.105**	-0.072*
	0.045	0.040
$(log)\text{Exports}_{e,t-1}$		-0.279***
		0.017
Industry Fixed Effects	Yes	Yes
Observations	3565	3565

*Notes: This table shows the results of the estimation of equation 4.3. All columns include industry fixed effects at the HS-2 digit level. All variables are standardized. Standard errors are reported under the estimated coefficients. Errors are clustered by sector at the HS-2 digit level. ***. ** and * denote significance at the 1%, 5% and 10% confidence levels.*

IV.D.3 Exporters that Switch their Importing Partner in Response to the Tariff Cuts Switch to More Productive Importers.

The next question is whether U.S. exporters respond to tariff cuts switching to more productive importing partners, as the model suggests. Answering this question requires information on the Colombian importing firms. For this purpose I use the balance sheet data on Colombian importers based on which I calculate a measure of the productivity of these importers, as I did in section II.

Once again, I rely on the variation across industries in the extent of tariff cuts. I estimate the following equation including only the subset of U.S. exporters that switch to a new partner during the liberalization. The dependent variable $\Delta\Theta_i^e$ is the percentage change in the productivity between the importing firm (i) trading with a given U.S. exporter (e) before and after the liberalization. The productivity of the initial and final importing partner is measured before the agreement in both cases, to prevent the concern that the productivity of the post-agreement importing partner is influenced directly by the tariff change. The independent variable is the exporter's exposure to the trade liberalization, captured by the tariff cuts relevant to each exporter.

$$\Delta\Theta_i^e = \beta \cdot \text{Tariff}_e + \epsilon_e \tag{4.5}$$

The number of observatios used for the estimation of equation 4.5 is small because it

only includes U.S. exporters i) with a single importing partner before and after the FTA ii) that switched to a different importing partner during the FTA and iii) exporters trading with Colombian partners included in the balance sheet data.

I compute the productivity of each Colombian importer as the residual of a regression of revenue on labor cost and industry fixed effects.[25] I include in the estimation of equation 4.5 fixed effects for the industry of the importing firm, at the same level of aggregation (3-digit ISIC codes) than those used when calculating the productivity of these importing firms.

The results are reported in table IV.6. Exporters that switch importing partners in response to the tariff cuts choose more productive importing partners. The estimated coefficient indicates that a one standard deviation larger tariff reduction is associated with 0.2 standard deviations higher productivity of the importer. The estimated coefficient is robust to including the productivity of the exporter's pre-liberalization importing partner ($\Theta^e_{i,PRE}$)(column 2) and the pre-liberalization exports of the exporter (column 3).

Table IV.6: Exporters that Switch their Importing Partner in Response to the Tariff Cuts Switch to More Productive Importers.

Dependent Variable: $\Delta\Theta^e_i$

	(1)	(2)	(3)
ΔTariff$_e$	-0.194*	-0.205*	-0.203*
	0.105	0.114	0.118
$\Theta^e_{i,PRE}$		-0.165	-0.171
		0.213	0.214
(log)Exports$_{e,t-1}$			-0.107
			0.109
Industry Fixed Effects	Yes	Yes	Yes
Observations	248	248	248

*Notes: This table shows the results of the estimation of equation 4.3. All columns include industry fixed effects at the ISIC-3 digit level which is the same level used when computing the productivity of Colombian importers. All variables are standardized. Standard errors are reported under the estimated coefficients. Errors are clustered by sector at the industry level. ***. ** and * denote significance at the 1%, 5% and 10% confidence levels.*

[25] Using balance sheet data on Colombian firms I estimate a proxy for productivity as the residual of $log(\text{sales})_i = \beta \cdot log(\text{labor costs})_i + \phi_p + \epsilon_i$, where i indexes firms and ϕ_p stands for industry (ISIC-3 digit) fixed effects.

To summarize, bringing together the findings of subsections IV.D.1, IV.D.2, and IV.D.3, these results indicate that tariff reductions to U.S. exporters lead them to increase their export sales, to switch importing partners, and to switch to more productive importing partners. This pattern is consistent with the theoretical model developed in this paper and portraits a new mechanism of adjustment of firms to trade liberalization, which highlights the importance of understanding exporter-importer relationships.

V. Conclusions

This paper has studied the interaction between exporting and importing firms, in the context of international trade based on increasing returns to scale with heterogeneous exporters and heterogeneous importers. Analyzing jointly the behavior of exporting and importing firms requires taking into account the selection of these firms into trading pairs. The paper proposes a simple model in which exporters search for importing firms, with whom they need to partner to reach final consumers abroad. The sorting of exporters and importers into trading pairs varies across destinations and across sectors, and has an influence on the pattern of trade. This represents a departure from existing models of international trade in which entry into export markets is a black box and the role of importing firms is ignored.

This paper has also provided empirical evidence in support of this theory. In the cross-section, I establish three new observations using matched exporter-importer data: i) most exporters trade with a single importer, ii) there is positive assortative matching between exporters and importers in terms of their productivity, and iii) the value traded is positively correlated with the productivity of both exporters and importers. Then, using the Colombia-US Free Trade Agreement as an exogenous shock I provide the first evidence on the reallocation of exporter-importer matches in response to trade liberalization. As predicted by the model, U.S. exporters facing larger tariff cuts in the Colombian market adjust by switching to more productive Colombian importers, which leads to an increase in their exports in addition to the direct effect of tariff cuts on the intensive margin.

It has been recently stressed that distribution networks or marketing channels could be important for understanding trade flows. Within the literature on heterogeneous firms in international trade, a recent survey by Redding (2011) suggests that an "area for further work is the microeconomic modeling of the trade costs that induce firm selection into export markets, including the role of wholesale and retail distribution networks." This paper moves in that direction by incorporating importing firms into the canonical heterogeneous-firms trade model.

This paper has stepped away from the typical assumption of frictionless international markets. In this model, finding partners in foreign markets is costly. This is an old idea, yet an underexplored one. In this regard, a set of general questions for further research is the following. Is the relatively high volume of trade experienced in the recent decades (the so-called second wave of globalization) a consequence of lower transport costs and tariffs or

a product of the increased access to information? Second, in their efforts to promote exports governments frequently subsidize information acquisition. Is this a reasonable policy?

Much is left for future research. One task left pending is to study bargaining in a context where a set of heterogeneous exporters are confronted to a set of heterogeneous importers. For instance, one could ask whether it is good or bad news when a small exporter - let's say a coffee farmer in a developing country - signs a contract with a very large wholesaler abroad (rather than with a smaller importer)? More generally, how are the gains from trade split between exporting and importing countries?

To conclude, one can use this model as a starting point to study other features of trade contracts. Exporting and importing firms in this model have to set prices and quantities, but trade contracts in practice are probably more complex. One can study other characteristics of these contracts, such as their duration, the invoice currency, vertical restraints, and financing terms, among others.

VII. References

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VIII. Empirical Appendix - For Online Publication

VIII.A Descriptive Statistics for Section II

The following table presents descriptive statistics for the dataset used in section II. These are descriptive statistics for the non-standardized variables. Productivity is shifted to have a mean zero. Coefficients in tables II.2 and II.3are based on these variables standardized to have mean 0 and standard deviation 1.

Table VIII.1: Descriptive Statistics for the data on French exporters and Colombian importers.

	Mean	Std. Deviation
Traded Value	10.235	2.223
Exporter - Revenue	17.415	2.041
Importer - Revenue	17.564	1.941
Exporter - Revenue minus bilateral trade	17.405	2.057
Importer - Revenue minus bilateral trade	16.899	1.958
Exporter - Estimated Productivity	0	0.642
Importer - Estimated Productivity	0	0.877

VIII.B Descriptive Statistics for Section IV.

The following table presents descriptive statistics for the dataset used in the estimation of equations 4.3, 4.4 and 4.5 in section IV. These are descriptive statistics for the non-standardized variables. Coefficients in tables IV.4, IV.5, and IV.6, are based on these variables standardized to have mean 0 and standard deviation 1.

Table VIII.1: Descriptive Statistics for Equation 4.3.

	Mean	Std. Deviation
$\Delta(log)\text{Exports}_e$	0.009	1.807
ΔTariff_e	-1.391	3.423
$(log)\text{Exports}_{e,t-1}$	10.134	2.130

Table VIII.1: Descriptive Statistics for Equation 4.4.

	Mean	Std. Deviation
Switch_e	0.250	0.433
ΔTariff_e	-1.391	3.423
$(log)\text{Exports}_{e,t-1}$	10.134	2.130

Table VIII.1: Descriptive Statistics for Equation 4.5.

	Mean	Std. Deviation
$\Delta\Theta_i^e$	-2.366	20.740
ΔTariff_e	-1.298	2.575
$\Theta_{i,PRE}^e$	0.582	1.024
$(log)\text{Exports}_{e,t-1}$	9.387	1.811

IX. Theory Appendix - For Online Publication

Subsections IX.A, IX.B and IX.C contain additional theoretical results. Subsections IX.D show the details of the solution of the partial equilibrium problem of the exporting firm. Subsection IX.E presents a variation in the case of Nash bargaining between exporters and importers. Subsection IX.F presents the solution to the general equilibrium model. Finally, subsection IX.G contains proofs to each proposition.

IX.A Wages and Welfare

Differences in country size lead to differences in wages. As in the two-country Melitz (2003) model with symmetric trade costs (see Felbermayr and Jung (2011)) it can be shown that:

Proposition IV. The larger country has a higher wage.

Proof: See subsection IX.G.

This result is equivalent to that in Krugman (1980) with homogeneous firms. Under costly trade, a wage differential is needed offset the attractiveness of the larger market to producers in order to keep labor fully employed in both markets.

Welfare is measured by the real wage in each country, $W_i = \frac{w_i}{P_i}$. As in the Melitz (2003) model, under costly trade welfare is higher in the larger country. In this case, this is found numerically, simulating the model for a wide variety of parameters.

IX.B Gravity in a Many-Country World

In partial equilibrium we can think of a many-country version of the model and derive a gravity equation that relates the volume of trade between two countries to their income and trade costs. The aggregate gravity equation derived from this model includes both the usual trade costs as well as search costs. Recall that Tinbergen's (1962) explanation for the role of distance in the inaugural gravity equation was not only an approximation for transport costs but also a representation of information frictions: "The factor of distance may also stand for an index of information about export markets".

Consider an M-country version of the model described earlier. Preferences in each country are identical, and transport costs and search costs vary across country pairs but are

symmetric ($\tau_{ij} = \tau_{ji}$ and $\lambda_{ij} = \lambda_{ji}$). There is a mass of exporting firms in each country (producers), which can also sell domestically. To derive the gravity equation of a many-country case, I can write the total exports from country j to country i by integrating across j's exporters. The value of total exports of country i to country j is then expressed as (to shorten notation, throughout this section I will use θ_E to denote the productivity of the exporters and θ_I the productivity of the importers):

$$R_{ij} = M_i \cdot \int_0^\infty \int_0^\infty R_E(\theta_E, \theta_I, Y_j) \cdot f_{max}^{n^*(\theta_E, Y_j, \lambda_{ij} \cdot w_i)}(\theta_I) f(\theta_E) d\theta_I d\theta_E = Y_j \cdot M_i \cdot \Gamma(\tau_{ij}, \lambda_{ij}, Y_j) \tag{9.1}$$

M_i is the number of exporting firms in country i and the function $\Gamma(\tau_{ij}, \lambda_{ij}, Y_j)$ is used to encompass a group of variables and parameters that includes trade costs, search costs and the income level of the importing country.

As in Helpman, Melitz and Rubinstein (2008), the fact that total income equals total expenditure in the exporting country is used to write the exporting country's income as the sum of the revenue from its exports to all countries (plus domestic sales). Using this equation, one can write the number of firms in the exporting country as a function of the exporting country's income level. The income in country i is equal to its exports to every country:

$$Y_i = \sum_h M_i \cdot \int_0^\infty \int_0^\infty R_E(\theta_E, \theta_I, Y_h) \cdot f_{max}^{n^*(\theta_E, Y_h, \lambda_{ih} \cdot w_i)}(\theta_I) f(\theta_E) d\theta_I d\theta_E$$

from where

$$M_i = \frac{Y_i}{\sum_h \Phi(\tau_{ih}, \lambda_{ih}, Y_h)} \tag{9.2}$$

The function $\Phi(\tau_{ij}, \lambda_{ij}, Y_h)$ is introduced to represent a number of variables and parameters. The next step is to replace the number of firms in the exporting country, M_i, back in equation (9.1) using (9.2). This leads to the resulting gravity equation that incorporates both standard ad-valorem trade costs as well as search frictions.

$$R_{ij} = Y_i \cdot Y_j \cdot \frac{\Gamma(\tau_{ij}, \lambda_{ij}, Y_j)}{\Phi(\tau_{ij}, \lambda_{ij}, Y_h)} \tag{9.3}$$

IX.C Final Prices and Border Prices

The model features a final price paid by consumers in the destination market, p_f, and the price of the transaction at the border (the "border price", "international price" or "wholesale price", p_w) paid by the importer to the exporter. The ratio between these prices is typically large and has been studied by Berger et al (2012) and others.[26]

Consider again the many country world of subsection III.J. In the model, this price ratio p_f/p_w varies across exporting firms and destinations. It also depends on industry characteristics.

Consider the variation in the expected price ratio for a given exporting firm across destinations, $E[p_f/p_w|\theta_E]$. Large search costs lead exporters to reduce their search intensity. This results in matches with low-productivity importers (i.e. a higher marginal cost for the distribution of the products) and the price ratio of border to retail prices p_f/p_w increases. Exporting firms that face high entry barriers in the Colombian market (for instance an exporter from Pakistan to Colombia which will likely face a large search cost parameter) have a large p_f/p_w ratio, while firms facing small entry barriers (a small search cost parameter) have a small p_f/p_w ratio.

IX.D Solution to the Producer's Problem

Producers maximize their expected profits choosing an optimal search intensity n. Consider the case of a producer in Home of productivity θ_{PH} searching for a distributor in Foreign. The maximization problem is the following:

$$max_n \int_0^\infty f^n_{max}(\theta_{DF})\pi^{Hf}_{PH}(\theta_{PH},\theta_{DF})d\theta_{DF} - w_H \cdot \lambda \cdot n$$

The first term in this equation represents the expected operating profits for the producer obtained from a trading relationship with a distributor of productivity θ_{DF}, given a search effort n. The integrand is the probability of being matched with a certain distributor times the operating profits that the producer obtains from the relationship. The second term is the cost of the search effort.

We can rewrite the maximization problem after replacing these terms as:

[26] Berger et al (2012) find that for U.S. imports the gap $(p_f - p_w)/p_w$ is on average in the range 50% - 70%. Berger et al (2012) measure p_f using the BLS' CPI microdata and p_w using the BLS' IPP microdata.

$$max_n \; Z^{Hf} \cdot \theta_{PH}^{\epsilon \cdot v - 1} \cdot \int_0^\infty \gamma \cdot n \cdot \theta_{DF}^{-\gamma - 1} \cdot e^{-n \cdot \theta_{DF}^{-\gamma}} \cdot \theta_{DF}^{\epsilon \cdot (1-v)} \cdot d\theta_{DF} - w_H \cdot \lambda \cdot n$$

To evaluate the integral, define $t = n \cdot \theta_{DF}^{-\gamma}$ and integrate in terms of t. The maximization problem is:

$$max_n \; Z^{Hf} \cdot \theta_{PH}^{\epsilon \cdot v - 1} \cdot \int_0^\infty n^b \cdot t^{-b} \cdot e^{-t} \cdot dt - w_H \cdot \lambda \cdot n$$

Note that

$$\int_0^\infty n^b \cdot t^{-b} \cdot e^{-t} \cdot dt = n^b \cdot \int_0^\infty t^{-b} \cdot e^{-t} \cdot dt = n^b \cdot R$$

with $R = \Gamma(1-b)$ and $b = \frac{\epsilon \cdot (1-v)}{\gamma}$.

The maximization problem is then

$$max_n \; (Z^{Hf} \cdot \theta_{PH}^{\epsilon \cdot v - 1} \cdot R \cdot n^b - \lambda \, w_H \, n)$$

the solution of which is

$$n^*(\theta_{PH}, Z^{Hf}, \lambda) = k \cdot \left(\frac{Z_{Hf} \cdot \theta_{PH}^{\epsilon \cdot v - 1}}{\lambda \cdot w_H} \right)^{\gamma / (\gamma - \epsilon \cdot (1-v))},$$

where $k = \left(\frac{\epsilon \cdot (1-v) \cdot \Gamma\left(\frac{\gamma - \epsilon \cdot (1-v)}{\gamma}\right)}{\gamma} \right)^{\gamma / (\gamma - \epsilon \cdot (1-v))}$

IX.E The Case with Nash-Bargaining between Exporters and Importers

In the model discussed in section III, producers set the wholesale price at which they sell to distributors. Based on this price, distributors choose the optimal final price. An alternative way of modeling the interaction between producers and distributors is such that these firms jointly set the final price and bargain over the surplus generated by the trading partnership. This case is briefly discussed below. The results described in section III still hold in this case. In the model below I assume all outside options are zero, so the share of surplus obtained by exporters is always the same. This is unrealistic, but accounting for the

selection of firms into trading pairs and the existence of outside options at the same time is a difficult issue beyond the scope of this paper.

Selling to consumers in Foreign requires production and distribution. Exporters must search for a distributor in Foreign. Once an exporter has found and chosen a distributor, the relationship generates profits $\pi_E(\theta_E, \theta_I)$ by producing and distributing the good to the consumers. These profits are split between both parties through Nash bargaining. The exporter receives a share $\beta \cdot \pi_E(\theta_E, \theta_I)$.

Exporters produce differentiated varieties and operate in a context of monopolistic competition. Consumers in Foreign have preferences over the set of varieties described by a utility function with constant elasticity of substitution α. The demand for a firm's variety is then

$$y = A \cdot p_f^{-\epsilon},$$

where $\epsilon = \frac{1}{1-\alpha} > 1$ and the term $A = \frac{I_F}{P_F^{1-\epsilon}}$ determined in equilibrium, combines income and the price index in Foreign.

Production and distribution are necessary to sell a variety to final consumers and are combined in Cobb-Douglas form. The joint cost of these activities is

$$cost = \left(\frac{w_E}{\theta_E}\right)^v \cdot \left(\frac{w_I}{\theta_I}\right)^{1-v}$$

Wages are w_E at Home and w_I at Foreign. Under this framework, the price charged to consumers is a constant markup over the cost, $p = cost/\alpha$. The maximized operating profits of the exporter - distributor relationship are

$$\pi_E(\theta_E, \theta_I) = A \cdot (1-\alpha) \cdot \left(\frac{1}{\alpha} \cdot \left(\frac{w_E}{\theta_E}\right)^v \cdot \left(\frac{w_I}{\theta_I}\right)^{1-v}\right)^{1-\epsilon}$$

Anticipating this, exporters choose their optimal search effort n, solving:

$$max_n \int_0^\infty f_{max}^n(\theta_I) \cdot \beta \cdot \pi_E(\theta_E, \theta_I) d\theta_I - w_E \cdot \lambda \cdot n$$

The first term in this equation represents the expected surplus for the exporter of a trading relationship with an importer θ_I, given a search effort n. The integrand is the probability of being matched with a certain importer times the operating profits that the exporter obtains from the relationship. The second term is the cost of the search effort.

IX.F Solution to the General Equilibrium

Consider first the zero profit conditions for producers in Home. In what follows θ_E represents the productivity of an exporter (producer) and θ_I the productivity of an importer (distributor). For notational simplicity I omit some of the subindices present in the main text.

$$\int_0^\infty \int_0^\infty \left(Z^{HH} \theta_E^{\epsilon\, v-1} \theta_I^{\epsilon\,(1-v)} - \lambda\, w_H n^*(\theta_E, Z^{HH}, \lambda\, w_H) \right) f_{max}^{n^*(\theta_E, Z^{HH}, \lambda\, w_H)}(\theta_I) f(\theta_E)\, d\theta_I\, d\theta_E +$$
$$\int_0^\infty \int_0^\infty \left(Z^{HF} \cdot \theta_E^{\epsilon\, v-1} \theta_I^{\epsilon\,(1-v)} - \lambda\, w_H n^*(\theta_E, Z^{HF}, \lambda\, w_H) \right) f_{max}^{n^*(\theta_E, Z^{HF}, \lambda\, w_H)}(\theta_I)\, f(\theta_E)\, d\theta_I\, d\theta_E$$
$$= w_H\ F_H \quad (9.4)$$

and for producers in Foreign:

$$\int_0^\infty \int_0^\infty \left(Z^{FF}\, \theta_E^{\epsilon\, v-1} \cdot \theta_I^{\epsilon\,(1-v)} - \lambda\, w_F\, n^*(\theta_E, Z^{FF}, \lambda\, w_F) \right) f_{max}^{n^*(\theta_E, Z^{FF}, \lambda\, w_F)}(\theta_I)\, f(\theta_E)\, d\theta_I\, d\theta_E +$$
$$\int_0^\infty \int_0^\infty \left(Z^{FH}\, \theta_E^{\epsilon\, v-1} \cdot \theta_I^{\epsilon\,(1-v)} - \lambda\, w_F \cdot n^*(\theta_E, Z^{FH}, \lambda\, w_F) \right) \cdot f_{max}^{n^*(\theta_E, Z^{FH}, \lambda\, w_F)}(\theta_I)\, f(\theta_E)\, d\theta_I\, d\theta_E$$
$$= w_F\ F_F \quad (9.5)$$

where

$$Z^{HH} = \frac{w_H \cdot L_H \cdot \left(\frac{\epsilon \cdot v}{\epsilon \cdot v - 1}\right)^{-\epsilon \cdot v} \cdot \left(\frac{\epsilon}{\epsilon - 1}\right)^{-\epsilon} \cdot \left(\frac{\epsilon}{\epsilon - 1}\right) \cdot w_H^{1-\epsilon \cdot v} \cdot w_H^{-\epsilon \cdot (1-v)}}{P_H^{1-\epsilon}}$$

$$Z^{HF} = \frac{w_F \cdot L_F \cdot \tau^{1-\epsilon v} \left(\frac{\epsilon \cdot v}{\epsilon \cdot v - 1}\right)^{-\epsilon \cdot v} \cdot \left(\frac{\epsilon}{\epsilon - 1}\right)^{-\epsilon} \cdot \left(\frac{\epsilon}{\epsilon - 1}\right) \cdot w_H^{1-\epsilon \cdot v} \cdot w_F^{-\epsilon \cdot (1-v)}}{P_F^{1-\epsilon}}$$

$$Z^{FH} = \frac{w_H \cdot L_H \cdot \tau^{1-\epsilon v} \left(\frac{\epsilon \cdot v}{\epsilon \cdot v - 1}\right)^{-\epsilon \cdot v} \cdot \left(\frac{\epsilon}{\epsilon - 1}\right)^{-\epsilon} \cdot \left(\frac{\epsilon}{\epsilon - 1}\right) \cdot w_F^{1-\epsilon \cdot v} \cdot w_H^{-\epsilon \cdot (1-v)}}{P_H^{1-\epsilon}}$$

$$Z^{FF} = \frac{w_F \cdot L_F \cdot \left(\frac{\epsilon \cdot v}{\epsilon \cdot v - 1}\right)^{-\epsilon \cdot v} \cdot \left(\frac{\epsilon}{\epsilon - 1}\right)^{-\epsilon} \cdot \left(\frac{\epsilon}{\epsilon - 1}\right) \cdot w_F^{1-\epsilon \cdot v} \cdot w_F^{-\epsilon \cdot (1-v)}}{P_F^{1-\epsilon}}$$

After some algebra, the zero profit conditions for producers in Home and Foreign can be written as:

$$Z^{HH^{\delta}} \cdot \int_0^{\infty} \int_0^{\infty} S \cdot \theta_E^{\delta \cdot (\epsilon \cdot v - 1)} \cdot \lambda^{\delta - 1} \cdot f(\theta_E) \cdot d\theta_E + Z^{HF^{\delta}} \cdot \int_0^{\infty} \int_0^{\infty} S \cdot \theta_E^{\delta \cdot (\epsilon \cdot v - 1)} \cdot \lambda^{\delta - 1} \cdot f(\theta_E) \cdot d\theta_E = w_H \cdot F_H \quad (9.6)$$

and for producers in Foreign:

$$Z^{FF^{\delta}} \cdot \int_0^{\infty} \int_0^{\infty} S \cdot \theta_E^{\delta \cdot (\epsilon \cdot v - 1)} \cdot \lambda^{\delta - 1} \cdot f(\theta_E) \cdot d\theta_E + Z^{FH^{\delta}} \cdot \int_0^{\infty} \int_0^{\infty} S \cdot \theta_E^{\delta \cdot (\epsilon \cdot v - 1)} \cdot \lambda^{\delta - 1} \cdot f(\theta_E) \cdot d\theta_E = w_F \cdot F_F \quad (9.7)$$

This is a linear system for $P_H^{\delta \cdot (\epsilon - 1)}$ and $P_F^{\delta \cdot (\epsilon - 1)}$. We obtain expressions for these price indices as functions of the wages. The terms S and δ group a number of parameters.

Next consider the equations for the price indices in Home and Foreign.

In Home:

$$P_H^{1-\epsilon} = M_H \cdot K \cdot w_H^v \cdot w_H^{1-v} \cdot \int_0^{\infty} \int_0^{\infty} \theta_E^{(\epsilon-1) \cdot v} \cdot \theta_I^{(\epsilon-1) \cdot (1-v)} \cdot f_{max}^{n^*(\theta_E, Z^{HH}, \lambda \cdot w_H)}(\theta_I) \cdot f(\theta_E) \cdot d\theta_I \cdot d\theta_E + M_F \cdot K \cdot w_F^v \cdot w_H^{1-v} \cdot \int_0^{\infty} \int_0^{\infty} \theta_E^{(\epsilon-1) \cdot v} \cdot \theta_I^{(\epsilon-1) \cdot (1-v)} \cdot f_{max}^{n^*(\theta_E, Z^{FH}, \lambda \cdot w_F)}(\theta_I) \cdot f(\theta_E) \cdot d\theta_I \cdot d\theta_E \quad (9.8)$$

In Foreign:

$$P_F^{1-\epsilon} = M_H \cdot K \cdot w_H^v \cdot w_F^{1-v} \cdot \int_0^{\infty} \int_0^{\infty} \theta_E^{(\epsilon-1) \cdot v} \cdot \theta_I^{(\epsilon-1) \cdot (1-v)} \cdot f_{max}^{n^*(\theta_E, Z^{HF}, \lambda \cdot w_H)}(\theta_I) \cdot f(\theta_E) \cdot d\theta_I \cdot d\theta_E + M_F \cdot K \cdot w_F^v \cdot w_F^{1-v} \cdot \int_0^{\infty} \int_0^{\infty} \theta_E^{(\epsilon-1) \cdot v} \cdot \theta_I^{(\epsilon-1) \cdot (1-v)} \cdot f_{max}^{n^*(\theta_E, Z^{FF}, \lambda \cdot w_F)}(\theta_I) \cdot f(\theta_E) \cdot d\theta_I \cdot d\theta_E, \quad (9.9)$$

where I have used $K = \frac{\epsilon}{\epsilon - 1} \cdot \left(\frac{\epsilon v}{\epsilon v - 1}\right)^v$.

The equations for the price index in Home and Foreign are a linear system in terms of the mass of producers in Home and Foreign, M_H and M_F. Given that we can find the price indices as functions of the wages from the zero profit conditions, we obtain also expressions for the mass of producers as functions of the wages.Since this is a linear system, this is solved

analytically.

Next, we replace these terms in the balanced trade condition, set one wage as the numeraire and solve for the other one. The balanced trade condition is the following.

$$M_H \cdot \int_0^\infty \int_0^\infty Z^{HF} \cdot \epsilon \cdot v \cdot \theta_E^{\epsilon \cdot v - 1} \cdot \theta_I^{\epsilon \cdot (1-v)} \cdot f_{max}^{n^*(\theta_E, Z^{HF}, \lambda \cdot w_H)}(\theta_I) \cdot f(\theta_E) \cdot d\theta_I \cdot d\theta_E =$$
$$M_F \cdot \int_0^\infty \int_0^\infty Z^{FH} \cdot \epsilon \cdot v \cdot \theta_E^{\epsilon \cdot v - 1} \cdot \theta_I^{\epsilon \cdot (1-v)} \cdot f_{max}^{n^*(\theta_E, Z^{FH}, \lambda \cdot w_F)}(\theta_I) \cdot f(\theta_E) \cdot d\theta_I \cdot d\theta_E \quad (9.10)$$

Finally the mass of importers in each country, N_H and N_F is obtained by replacing the solution into the zero profit condition of importers. A useful property of the model is that the mass of importers does not affect the other equilibrium variables.If search were modeled differently, this would not hold.

The zero profit condition for importers in Home is the following. J denotes the cost of entry into importing.

$$\int_0^\infty \int_0^\infty f_{max}^{n^*(\theta_E, Z^{HH}, \lambda \cdot w_H)}(\theta_I) \cdot \frac{1}{N_H \cdot f(\theta_I)} \cdot B_{HH} \cdot \theta_E^{(\epsilon-1) \cdot v} \cdot \theta_I^{(\epsilon-1) \cdot (1-v)} \cdot M_H \cdot f(\theta_E) \cdot f(\theta_I) \cdot d\theta_I \cdot d\theta_E +$$
$$\int_0^\infty \int_0^\infty f_{max}^{n^*(\theta_E, Z^{FH}, \lambda \cdot w_F)}(\theta_I) \cdot \frac{1}{N_H \cdot f(\theta_I)} \cdot B_{FH} \cdot \theta_E^{(\epsilon-1) \cdot v} \cdot \theta_I^{(\epsilon-1) \cdot (1-v)} \cdot M_F \cdot f(\theta_E) \cdot f(\theta_I) \cdot d\theta_I \cdot d\theta_E = w_H \cdot J_H \quad (9.11)$$

In Foreign:

$$\int_0^\infty \int_0^\infty f_{max}^{n^*(\theta_E, Z^{HF}, \lambda \cdot w_H)}(\theta_I) \cdot \frac{1}{N_F \cdot f(\theta_I)} \cdot B_{HF} \cdot \theta_E^{(\epsilon-1) \cdot v} \cdot \theta_I^{(\epsilon-1) \cdot (1-v)} \cdot M_H \cdot f(\theta_E) \cdot f(\theta_I) \cdot d\theta_I \cdot d\theta_E +$$
$$\int_0^\infty \int_0^\infty f_{max}^{n^*(\theta_E, Z^{FF}, \lambda \cdot w_F)}(\theta_I) \cdot \frac{1}{N_F \cdot f(\theta_I)} \cdot B_{FF} \cdot \theta_E^{(\epsilon-1) \cdot v} \cdot \theta_I^{(\epsilon-1) \cdot (1-v)} \cdot M_F \cdot f(\theta_E) \cdot f(\theta_I) \cdot d\theta_I \cdot d\theta_E = w_F \cdot J_F \quad (9.12)$$

In these equations,

$$B_{ij} = \frac{w_j \cdot L_j \cdot \left(\frac{\epsilon \cdot v}{\epsilon \cdot v - 1}\right)^{1-\epsilon \cdot v} \cdot \left(\frac{\epsilon}{\epsilon - 1}\right)^{-\epsilon} \cdot \left(\frac{\epsilon}{\epsilon - 1}\right) \cdot w_i^{1-\epsilon \cdot v} \cdot w_j^{1-\epsilon \cdot (1-v)}}{P_j^{1-\epsilon}}$$

The mass of importers in each country is obtained from these equations once the other equilibrium variables have been determined.. This can be done analytically.

This ends the solution to the general equilibrium.

IX.G Proofs to Propositions in section III.

Proof to Proposition I.

That more productive search more is seen by inspection of exporters' optimal search intensity (3.6). Under the restrictions imposed on the parameters, it is verified that the derivative of $n*$ with respect to θ_{PH} is positive.

Proof to Proposition II.

From the expression for an exporter's optimal search intensity, one can see that the search cost parameter λ impacts the exporter's decision both directly and through the price index. From the solution to the general equilibrium, as described in the previous section, one can write the price indices as functions of the wages and the search cost parameter. Under symmetric trade costs (the case in which I focus), the search cost parameter does not affect the price indices through the wages, only through the direct effect. In this way one can directly calculate the derivative of the price index to the search cost parameter, which is then used to calculate the derivative of the exporter's optimal search intensity with respect to the search cost parameter.

From the explicit solution for P_F and P_H it follows that both P_H and P_F depend on λ as

$$P \approx \lambda^{\frac{1-\delta}{\delta(1-\epsilon)}},$$

where $\delta = \gamma_i/(\gamma_i - \epsilon(1-v))$. From the expression for n^* we know that $n^* \approx (Z/\lambda)^\delta$. Since Z depends on λ through the price indices we obtain that n^* is proportional to $1/\lambda$. Therefore $dn^*/d\lambda < 0$.

Proof to Proposition IV.

I can write the balanced trade condition as a relationship between the relative wage $y = \frac{w_F}{w_H}$ and relative country sizes L_H/L_F as

$$\frac{L_H}{L_F} = H(y)$$

where

$$H(y) = \frac{X(y)^{(1+\delta c)/\delta}}{y^{\delta c}} \frac{\left[1 + \frac{y^r(\tau y)^p}{X(y)}\right]}{\left[(\tau/y)^p + \frac{y^r}{X(y)}\right]}$$

with

$$X(y) = \frac{1 - \tau^\delta (y/\tau)^{\delta\epsilon\, v}}{1 - \tau^\delta (y\tau)^{-\delta\epsilon\, v}},$$

$$p = v(1-\epsilon) + \delta c(1-\epsilon\, v)$$

and

$$r = \delta[(b-c) + \epsilon(2\, v - 1)] - \delta(1-\epsilon).$$

I can prove that the relationship between relative country size and relative wages is monotonic, that identical countries have identical wages, and that the larger country will have a higher wage.

To prove this, I show that $H(y)$ is a decreasing function of y. The steps of the proof are the following:

First, by taking its derivative we prove that $X(y)$ decreases monotonically with y. Next verify that defining $\mathcal{Z} \equiv y^{(r+p)}/X(y)$ and $\mathcal{Q} \equiv (1+\tau^p \mathcal{Z})/(\tau^p + \mathcal{Z})$ $H(y)$ can be written as

$$H(y) = \frac{X(y)^{(1-\frac{b}{\epsilon})}}{y^{\delta\, v(\epsilon-1)}} \mathcal{Q} \tag{9.13}$$

where we used that

$$\frac{1+\delta c}{\delta} = 1 - \frac{b}{\epsilon}, \qquad \delta c - p = \delta\, v(\epsilon - 1).$$

Next, verify that $\mathcal{Z}$ increases with y, and that $\mathcal{Q}$ decreases with y.

Finally we see that the above implies that $H(y)$ decreases with y. Effectively, since $X(y), \mathcal{Q}$ and y are positive and, since $1 - b/\epsilon > 0$ the terms in the numerator of (9.13) decrease with y and the term in the denominator increases with y (because $\epsilon - 1 > 0$). Therefore $H(y)$ decreases with y.